

This task was developed by secondary mathematics and CTE teachers across Washington State from urban and rural areas. These teachers have incorporated financial literacy in their classroom and have received training on the Common Core State Standards and the Jump\$tart Financial Literacy standards. The task was validated by content experts in the Common Core State Standards in mathematics. The purpose of this task is to demonstrate how financial literacy standards can be incorporated within mathematics classrooms that are implementing the Common Core State Standards.

TASK: Present Value Analysis

TARGET COMMON CORE STATE STANDARD(S) IN MATHEMATICS:
F-IF.B: Interpret functions that arise in applications in terms of the context
F-LE.A: Construct and compare linear, quadratic, and exponential models and solve problems
F-LE.A.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
TARGET STANDARDS FOR MATHEMATICAL PRACTICES:
MP 1: Making sense of problems and persevere in solving them
MP 2: Reason abstractly and quantitatively
MP 4: Model with mathematics
MP 5: Using appropriate tools strategically
MP 6: Attend to precision
TARGET FINANCIAL LITERACY STANDARDS:
Financial Responsibility and Decision Making:
1: Take responsibility for personal financial decisions,
4: Make financial decisions by systematically considering alternatives and consequences,
5: Develop communication strategies for discussing financial issues.
Planning and Money Management:
3: describe how to use different payment methods and
4: Apply consumer skills to purchase decisions
Credit and Debt:
1: Identify costs and benefits of various types of credit
RECOMMENDED COURSE(S):
Algebra 1, Personal Finance
ADDITIONAL INSTRUCTIONS:
Find an online credit card minimum payment calculator that the students can use for this task http://money.msn.com/credit-cards/credit-card-payoff-calculator.aspx (one example)
And an online installment loan payoff calculator that the students can use for this task http://www.bankrate.com/calculators/mortgages/loan-calculator.aspx (one example)
Additional readings for teachers and students: http://www.richcreditdebtloan.com/present-value-and-future-value-understanding-the-difference/
This task should be given after students learn about APR, Average Daily Balance, present value, loan length formula and loan payment formulas.

Common Core State Standards in Mathematics

The Common Core State Standards (CCSS) is a state developed set of standards that represent a coherent progression of learning expectations in English language arts and mathematics. These standards are designed to establish a set of shared goals and expectations for what students should understand and be able to do in grades K–12 in order to be prepared for success in college and the workplace. Forty-six states have now adopted these shared standards. The CCSS for mathematics highlight three major shifts around Focus, Coherence and Rigor. For more information:

<http://www.k12.wa.us/CoreStandards/Mathematics/default.aspx>

JumpStart Financial Literacy Standards

The National Standards in K–12 Personal Finance Education, created and maintained by the JumpStart Coalition® for Personal Financial Literacy, delineate the personal finance knowledge and skills that K–12 students should possess. The JumpStart Coalition intends the National Standards in K–12 Personal Finance Education to serve as a model. As such, the National Standards represent the framework of an ideal personal finance curriculum, portions of which might not be appropriate for individual instructors and students. The Coalition leaves it up to various stakeholders to decide how to address the topics in the National Standards. <http://jumpstart.org/national-standards.html>

Materials and tools:

Calculator

Computer with internet access

Teacher background reading and student reading:

Overview of Mathematics and Economics:

It is common for a high school student to receive multiple invitations to enroll for a credit card. In fact, an increasing number of high school students even carry credit cards. One goal of this lesson is to try to uncover some of the mathematics that underlies the calculation of numbers that are found on a monthly credit card statement. While these calculations are done using technology, it will likely help students understand the cost of maintaining balances on a credit card if they are to work out the numbers on their own. The first part of this lesson looks at the computation of the monthly balance on a credit card after calculating monthly finance charges. It illustrates how hard it is to pay down a credit card balance when finance charges are carried from month to month and the minimum monthly payment is made. It also illustrates how paying down a credit card balance becomes even more difficult when the card is used to obtain additional credit from one month to the next. Students also explore the benefits of making payments above the minimum payment and the mathematical structure that leads to credit card debt.

How is the monthly payment on an installment loan calculated? When a student visits a bank to borrow funds to purchase a used auto, the loan officer simply informs the student what the monthly payment will be. The same occurs for a fixed-rate mortgage loan as well as a student or debt consolidation loan. The common characteristic of each of these loans is that a fixed payment is made each month until the entire loan is paid off. The economic concept that underlies the mathematical calculation of a fixed payment has to do with present value analysis—the notion that one dollar received today is more valuable than a dollar received one year from today. In the last part of this lesson, students discover how the loan officer obtains



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the number that borrowers are given when the monthly installment on a fixed payment loan is calculated. The computation made by the student is (with rounding) equal to that which is found through using technology to arrive at a mechanical calculation.

Understanding where numbers (such as credit card finance charges and the computation of a monthly installment payment) come from can be very empowering for students. It will help them explain these relationships to others who don't understand the mathematics underlying complex financial calculations. It will also help keep them from having undesirable charges (such as optional payment insurance and mistaken card charges) added to loans and credit card balances. This makes students less vulnerable to unscrupulous finance officers and impersonal credit card companies. An ability to visualize the underlying mathematics and economics of financial calculations such as those demonstrated in this lesson will certainly serve students well in their personal and professional lives.

The mathematics used in this lesson ranges from simple computations of mathematical averages to more complex calculations of the sum of multiple degree polynomials. PART A of this lesson (on credit card economics) can be taught within pre-algebra classes on up. PART B of this lesson (on auto installment loan) can be taught from Algebra on up. Business teachers may also find this lesson to be useful.

Adapted from Lesson 15 from Mathematics and Economics: Connections for Life © National Council on Economic Education, New York, NY

Key Terms

Average Daily Balance
APR
Compounding Interest
Present value
Installment loan
Future value

The Task

Explore credit card debt and installment loan terms in order to explain the mathematics that underlie the mathematical calculations and patterns of monthly finance charges, monthly balance on a credit card, and calculation of a fixed payment on an installment loan.

Part A

The ending balance on your credit card is \$1516 and you decided to put your credit card away. You still have a limited income so you are constrained to making the 2% minimum monthly payment with an interest rate (APR) of 21%.

1. Use the table below to determine your monthly credit card balance for 5 months.
2. Show how you calculated each of the values for Month 1.

	Month 1	Month 2	Month 3	Month 4	Month 5
*Ave Daily Balance	\$1516.00				
**Mthly Finance Charge					
Total					
Min. Payment (2%)					
New Balance					

* Use your New Balance as the Ave Daily Balance for the next month.

**** Monthly Finance Charge Formula** (use 30 days for each billing cycle)—

$$\left[\frac{\text{days in billing cycle}}{365} \times \text{interest rate} \right] \times \text{Ave Daily Balance}$$

Use an online credit card calculator to answer the following questions:

3. How long would it take you to pay off your credit card and how much will you pay in interest if you only made your minimum monthly payment?

Pay off time _____ Amount of interest _____

4. The amount of interest incurred and the time to pay off the credit card is extremely high. What mathematical growth pattern or structure (linear, quadratic, exponential, etc.) leads to this high interest?

5. Explain how this growth pattern /structure leads to such high interest costs and payoff time.

6. If you paid an additional \$50 a month to your credit card, how long will it take to pay off your credit card and how much will you pay in interest?

Pay off time _____ Amount of interest _____

7. Explain why making a payment greater than the minimum payment significantly reduces the total interest paid and the length of time to pay off the credit card. Use data from your online calculator and your understanding of the mathematics to explain this.

8. Would it financially be more beneficial to lower your credit card interest rate to 15% or to make an additional \$50 payment each month? Use mathematics to justify your answer.

Part B

You have just decided to buy a used automobile, but will need to borrow \$5000 from the bank to complete your purchase. You have already spoken with the bank loan officer about the terms of the loan. The terms are: \$5000 loan made at a 10% rate of interest (APR), to be repaid in equal monthly installments over 3 years.

9. a. What is the value of the monthly fixed payment on your auto loan? _____

The monthly payment can be found by using the following formula:

$$P = \frac{Pv * \frac{r}{12}}{1 - \left(1 + \frac{r}{12}\right)^{-n}}$$

Where:

- P = Monthly Payment
 Pv = Present Value (beginning value or amount of loan)
 r = Interest rate (as a decimal)
 n = # of interest periods for overall time period (i.e., interest periods per year * number of years)

- b. Show the steps you used to solve this problem.

Use an online installment loan payoff calculator to double check your calculations from question #8 and to answer the following questions:

- c. How much will you pay in interest? _____
10. a. If you paid an additional \$50 a month to your auto loan, how long would it take you to pay it off? _____
- b. How much will you pay in interest? _____

11. Why is it important to understand the underlying mathematics for computing credit card debt and installment loans?

Possible Solutions

1.

	Month 1	Month 2	Month 3	Month 4	Month 5
*Ave Daily Balance	\$1516.00	\$1511.33	\$1506.67	\$1502.03	\$1497.40
**Mthly Finance Charge	\$26.17	\$26.09	\$26.01	\$25.93	\$25.85
Total	\$1542.17	\$1537.42	\$1532.68	\$1527.96	\$1523.25
Min. Payment (2%)	\$30.84	\$30.75	\$30.65	\$30.56	\$30.47
New Balance	\$1511.33	\$1506.67	\$1502.03	\$1497.40	\$1492.78

2. Students should show, mathematically, they arrived at the values within the table.
3. Pay off time: about **33.5 years** Amount of interest: about **\$6410.00**
4. Students should have an understanding that the structure is exponential growth.
5. Students should explain why this is exponential growth.
6. Pay off time: about **2.3 years** Amount of interest: about **\$388.00**
7. Students should use data from the online calculator to support their explanation.
8. Would it financially be more beneficial to lower your credit card interest rate to 15% or to make an additional \$50 payment each month? Use mathematics to justify your answer.
Much more beneficial to make an additional \$50 payment per month. \$50/month = about \$400 in interest. Lowering interest rate to 15% = about \$1700 in interest.
9. a. What is the value of the monthly fixed payment on your auto loan? **\$161.34**
 b. How much will you pay in interest? **\$808**
10. a. If you paid an additional \$50 a month to your auto loan, how long would it take you to pay it off? **2 yrs 3 months**
 b. How much will you pay in interest? **\$592**
11. Why is it important to understand the underlying mathematics for computing credit card debt and installment loans?

Possible Extensions

If you increased the additional amount you paid per month to \$100, \$125, etc...how would this affect the length of time it would take to pay off your credit card?

If you increased the additional amount you paid per month to \$100, \$125, etc...how would this affect the length of time it would take to pay off your installment loan?